On the Effectiveness of Recurrent Methods of Parameter Estimation in Macromodeling Problems

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Abstract — **Effectiveness of model parameter estimation using a recurrent bordering method is considered. By a test example, this algorithm is shown to be the faster in the problem of structure and parameter identification.**

Index terms — **macromodeling, parameter estimation, bordering method.**

A. PREFACE

The macromodeling problem [1] consists in building a mathematical model restoring a relationship between the given sets of input and output signals of a plant being modeled. The structure and parameter identification problem being solved in control theory has the same content. In a macromodeling problem, parameter estimation stage is the most resource demanding one. The choice of estimation method enhancing the processing speed requires further research.

B. INTRODUCTION

When using the least-squares method, the estimation problem is reduced to solving a system of linear algebraic equations.

This problem has some algorithmic aspects influencing significantly the estimation quality.

For solving a system of linear algebraic equations, the most known methods are the following:

– Gauss-Jordan elimination method,

– Kraut method, and

– Gramm-Schmidt orthogonalization method.

As it is known [2], the solutions by these methods are equivalent in the case of well-conditioned design matrix.

A recurrent algorithm of parameter estimation is described in [3] on the basis of well-known bordering method [4]. The idea of this algorithm consists in step-by-step improving estimations of parameters using recurrent calculation of inverse matrix elements in an estimation procedure. The efficiency analysis of the bordering method was done in [4] showing that this method is equivalent to classical methods according to number of operations and results.

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The recurrent bordering algorithm is in details described in the section C of the article.

The section D is devoted to consideration of some important features of the algorithm.

In the section E by a test example it is shown that bordering method gives essential advantage in performance time when solving a problem of structural-parametrical identifcation.

Some conclusions are given in the section F.

C. RECURRENT ESTIMATION ALGORITHM

Computing formulas for parameter estimation by the bordering method are given below.

Given the sample $W = (X \vdots y)$ of dimension $\dim W = n \times (m+1)$, the system of normal equations follows as:

$$X^T X\theta = X^T y. \tag{1}$$

Extended system matrix is:

$$\left[X^T X \vdots X^T y\right]. \tag{2}$$

Least-squares method gives the following parameter estimation:

$$\hat{\theta} = \left(X^T X\right)^{-1} X^T y. \tag{3}$$

Let for the general system matrices be new designations:

$$H \stackrel{\Delta}{=} X^T X = \left\{H_{ij}, \quad i, j = \overline{1, m}\right\}, \tag{4}$$

$$g \stackrel{\Delta}{=} X^T y = \left\{g_i, \quad i = \overline{1, m}\right\}. \tag{5}$$

We introduce designations, which conform to the model containing (s-1) arguments:

$$X_{s-1} \stackrel{\Delta}{=} [x_1 \vdots x_2 \vdots \ldots \vdots x_{s-1}],$$

$$X_{s-1}^T X_{s-1} \stackrel{\Delta}{=} H_{s-1} \qquad X_{s-1}^T y \stackrel{\Delta}{=} g_{s-1} \tag{6}$$

With inclusion of s-th argument, we may write down the normal system matrices in the form:

$$H_s = \begin{bmatrix} H_{s-1} & h_s \\ h_s^T & \eta_s \end{bmatrix}, \; g_s = X_s^T y = \begin{bmatrix} g_{s-1} \\ \gamma_s \end{bmatrix} \tag{7}$$

$$h_s = X_{s-1}^T x_s\,,\quad \eta_s = x_s^T x_s\,,\quad \gamma_s = x_s^T y\,. \qquad (8)$$

When including a next argument into the model, it is necessary to calculate a new inverse matrix:

$$H_s^{-1} = \begin{bmatrix} H_{s-1}^{-1} + \frac{1}{\beta_s} a_s a_s^T & -\frac{1}{\beta_s} a_s \\ -\frac{1}{\beta_{s+1}} a_s^T & \frac{1}{\beta_s} \end{bmatrix}, \qquad (9)$$

where

$$a_s = H_{s-1}^{-1} h_s\,, \qquad H_0^{-1} \overset{\Delta}{=} 0\,, \qquad (10)$$

$$\beta_s = \eta_s - h_s^T a_s\,; \qquad (11)$$

$$\hat{\omega}_s = \frac{1}{\beta_s}(\gamma_s - h_s \theta_{s-1})\,, \qquad (12)$$

$$\theta_s = \begin{bmatrix} \theta_{s-1} - \hat{\omega}_s a_s \\ \hat{\omega}_s \end{bmatrix},\ \hat{\theta}_0 \overset{\Delta}{=} 0\,. \qquad (13)$$

As it will be shown further it is advisable to use the bordering method for structure and parameter identification in a class of nested structures. Note that it can be used for solving linear algebraic equations systems in the case of a symmetric matrix as well.

D. THE ALGORITHM FEATURES

One can specify some features of the bordering algorithm allowing to receive additional run-time information.

1. Components of the vector a_s in (10) are regression coefficients for

$$x_s = \varphi(x_1, x_2, \ldots, x_{s-1})\,, \qquad (14)$$

i.e. they determine dependence of an argument, included in the s-th step, on arguments included in the model previously.

2. Coefficient β_s is simultaneously the mean-square error for the model (14), or the residual sum of squares $RSS(s)$.

3. The coefficient β_s is some characteristic of a conditionality degree of the information matrix X^TX. If the degree approaches to zero the inclusion of s-th argument will appreciably increase the matrix condition number and consequently will impair the solution accuracy. It may be recommended to check whether β is less than the limit value and to exclude the appropriate argument from the model if it is the case.

4. Expression

$$R_s = 1 - \frac{\beta_s}{\kappa_s}\,,\ \text{where}\ \ \kappa_s = \sum_{i=1}^{n} (x_{si} - \bar{x}_s)^2\,, \qquad (15)$$

is the coefficient of multiple correlation between vector x_s and vectors $x_1, x_2, \ldots, x_{s-1}$, and therefore can be used for estimating how intimate that correlation is.

Formulas (9) - (11) correspond to the recurrent algorithm for calculation of inverse matrix.

E. NUMERICAL RESULTS

To check effectiveness of the bordering algorithm let us compare time of performance of structure and parameter identification in the nested structures class by a test example. The four parameter estimation methods were applied:

Gauss-Jordan elimination method,

Kraut method,

Gram-Schmidt orthogonalization method, and

recurrent bordering method.

A matrix X was generated of the size 290×300 (300 equations with 290 unknowns). The condition number of the matrix X was 4.6e+006 (i.e. relatively small one).

Table shows the obtained results.

Table 1. Run time (processor Intel Celeron 800)

Method	**Runtime, s**
Gram-Schmidt	101
Gauss-Jordan	92
Kraut	63
Bordering method	5.7

As it is evident from the table, the bordering algorithm has the least run time. It has to be noted here that accuracy of all the methods was identical.

F. CONCLUSIONS

For a modeling problem of high dimensionality (regressors number m=290, points number n=300), it is shown that recurrent bordering algorithm has considerably higher computation speed than other algorithms. Thus, use of the algorithm is effective for parameter estimation in the problem of modeling from observation data.

It is worthy to note, that such a recurrent algorithm is optimal not only for sequential estimation of model parameters in the nested structures class but also in the general case of exhaustive search of all possible models structures [3].

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